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TEMPERATURE AND COOLANT-INDUCED REACTIVITY VARIATIONS AND THEIR RELATION TO CONTROL WORTH IN NUCLEAR ROCKET REACTORS

by Paul F. Herrmann

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Cleveland, Ohio

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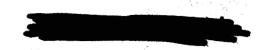
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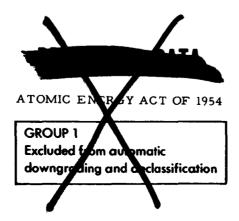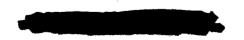
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TEMPERATURE AND COOLANT-INDUCED REACTIVITY VARIATIONS

AND THEIR RELATION TO CONTROL WORTH IN

NUCLEAR ROCKET REACTORS (U)

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SUMMARY

In graphite-core nuclear reactors for rocket propulsion applications, the most significant reactivity variations occur during the power ascension. These variations are attributable to the introduction of gaseous hydrogen coolant (propellant) into the core and reflector and to the increasing temperature and consequent expansion of the core.

In this report, the magnitude and sign of these effects were investigated, and they were compared with the worth of the peripheral control systems to determine which reactors have adequate control worth. The effect of full-power operating conditions on the worth of the control systems was also investigated. Reactors with a wide range of core void fractions (0.2 to 0.4) and fuel concentrations (the ratio of moderator (carbon) to fuel (uranium 235) atoms ranged from 50 to 5000) were included. The cores were unreflected axially with a height of 132.08 centimeters (52 in.). They had a graphite inner support annulus that was 5.08 centimeters (2 in.) thick and an annular beryllium reflector with a void fraction of 0.175 and a thickness of 11.43 centimeters (4.5 in.).

Controllability imposes no serious restriction on the choice of a reactor design. For a shutdown margin of at least 2 dollars, reactors with diameters of up to 180 to 190 centimeters and flow passage areas of 0.9 to 1.4 square meters (10 to 15 ft^2) can be controlled. Power operation has the effect of increasing the worth of the control systems slightly - in most cases, by less than a dollar.

INTRODUCTION

The nuclear reactor configuration that has received the most attention for rocket propulsion applications is cylindrical with a core that consists of a graphite matrix containing uranium carbide with the uranium enriched to 93 percent in uranium 235. An annulus of
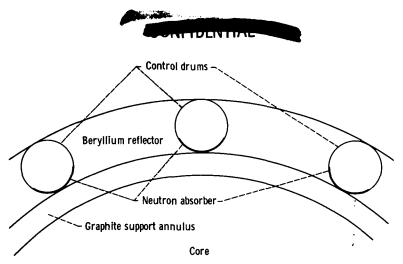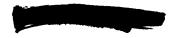
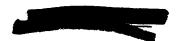Figure 1. - Partial cross section of nuclear rocket reactor.

graphite or other structural material, possibly in combination with the reflector, houses a core support system. A beryllium reflector several inches thick surrounds the graphite annulus and houses a number of core-height control drums. The drums rotate on an axis parallel to the vertical axis of the reactor. Along their length and over a fraction of their circumference they are coated with a material that is a strong neutron absorber. A partial cross section of the reactor is shown in figure 1.

During the power ascension of these reactors, variations in temperature, density, dimensions, and composition change the effective multiplication factor k_{eff}. However, the purpose of a reactor control system is to maintain the effective multiplication factor at or near unity at all times. Therefore, from the point of view of controllability, the feasibility of constructing a reactor depends on whether or not the worth of the control system exceeds the range over which the k_{eff} can be expected to change during the operation of the reactor. Control-system worth is obtained from the difference between the maximum value of k_{eff} when the absorber material is farthest from the core and the minimum value when it is closest to the core.

A parametric study to determine the effect of core void fraction, ratio of moderator (carbon) to fuel (uranium 235) atoms, and other geometrical and structural parameters on the control-system worth was made by Peoples and Fieno (ref. 1). A radial, one-dimensional, multigroup, multiregion diffusion code was used to determine the core diameter required to achieve a k_{eff} of 1.05 and then to calculate the worth of the control system. The present study was conducted to determine the magnitude and sign of the reactivity perturbation induced by the power ascension and the introduction of gaseous hydrogen coolant (propellant) into the core and reflector. These results were then compared with control-system worth to determine which reactors have adequate control worth. The study was also conducted to determine what effect, if any, full-power operating conditions have on the worth of the control system.

These effects are investigated for reactors of the same general type and description as those discussed on pages 7 to 9 of reference 1. These reactors are unreflected axially

with a height of 132.08 centimeters. The core void fractions range from 0.2 to 0.4, and the moderator to fuel ratio from 50 to 5000. The graphite core-support annulus is 5.08 centimeters thick, and the beryllium reflector is 11.43 centimeters with a void fraction of 0.175.

CALCULATION METHODS AND PARAMETERS

Reactor calculations were made by using a one-dimensional, multigroup, multiregion diffusion code (RP-1) that was written for use on the Lewis 7094 digital computer system. The General Atomics code, GAM II (ref. 2), was used to provide diffusion coefficients and macroscopic cross sections for the five broad groups into which the fast portion of the neutron energy spectrum was divided. An Atomics International code, TEMPEST (ref. 3), supplied one-group data that was averaged over the thermal part of the spectrum from the GAM II energy minimum, 0.414, down to 0 electron volt. The group energy boundaries are listed in table I.

For each of the 20 combinations of the core void fraction and the moderator to fuel ratio that was included, the radius of the core was calculated such that the k_{eff} of the reactor was 1.05. (Symbols are defined in appendix A.) An excess multiplication of 0.05 was considered an adequate margin of reactivity to compensate for coolant and temperature effects, for subsequent addition of structural components, cladding, and control drums, and also for shimming. These calculations were made with the coolant passages void of hydrogen and with the reactor system at room temperature, 293° K. Atom densities of carbon, uranium 235, and uranium 238 (table II) were calculated by using the equations presented in appendix B. Natural atom densities of graphite and beryllium were 0.0803 and 0.1236 atom per barn-centimeter, which correspond to material densities of 1.6 and 1.85 grams per cubic centimeter, respectively.

Hydrogen at 293° K and a pressure of 1 atmosphere was then introduced into the core and reflector. Appendix B also shows the calculation that converts gas densities to atom densities. The calculation of the k_{eff} and the worth of hydrogen to the reactor was made by using the same atom densities of core and reflector materials and the same dimensions. The calculations of reactivity worth are shown in appendix C.

By using temperatures, pressures, atom densities, and reactor dimensions appropriate to four levels of reactor power k_{eff} and the worth of the combined effects of hydrogen, high temperature, and expansion were calculated. ("Levels of reactor power" refers to percentages

TABLE I. - RANGE OF NEUTRON

ENERGY GROUPS

Group	Energy, eV
1	14.9×10^6 to 2.23×10^6
2	2.23×10^6 to 8.21×10^5
3	8.21×10^5 to 5.53×10^3
4	5.53×10^3 to 61.4
5	61.4 to 0.414
6	0.414 to 0
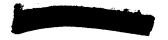
TABLE II. - ATOM DENSITIES OF CORE MATERIALS

[Temperature, 293° K.]

Moderator to fuel ratio, R	Core void fraction, α	Atom density, atoms/b-cm		
		Carbon	Uranium 235	Uranium 238
50	0.2	0.0628	0.00126	0.0000948
	.3	.0550	.00110	.0000828
	.4	.0471	.000942	.0000709
100	0.2	0.0636	0.000636	0.0000479
	.3	.0556	.000556	.0000419
	.4	.0477	.000477	.0000359
200	0.2	0.0638	0.000319	0.0000240
	.3	.0558	.000279	.0000210
	.4	.0478	.000239	.0000180
500	0.2	0.0640	0.000128	0.00000963
	.3	.0560	.000112	.00000843
	.4	.0480	.0000960	.00000723
1000	0.2	0.0642	0.0000642	0.00000483
	.3	.0561	.0000561	.00000422
	.4	.0481	.0000481	.00000362
2000	0.2	0.0642	0.0000321	0.00000242
	.3	.0561	.0000280	.00000211
	.4	.0481	.0000240	.00000181
5000	0.2	0.0642	0.0000128	0.000000963
	.3	.0561	.0000112	.000000843

of full power rather than to absolute magnitudes. A particular percentage of full power will correspond approximately to the same values of temperature and pressure for the entire group of reactors, although, because of differences in core size, the power in MW will differ.)

At full power, the room-temperature core atom densities were reduced by 2 percent to account for bulk thermal expansion at an average core material temperature of 1890° K. This compares to 1.9 percent (1830° K) from a comparison of atom densities at room and operating temperatures for the NRX-A reactor on pages 2-13 to 2-20 of reference 4. The temperature of the graphite support annulus at full power was 600° K, and the atom density reduction was 0.5 percent. In the reflector, the temperature was 150°K, and the atom density increased (thermal contraction) 0.3 percent (ref. 5). For the diffusion code, the radial and axial dimensions of the core were increased by 0.75 (bulk ex-

pansion of a cylinder is proportional to the square of the radial component) and 0.5 percent, respectively. The radial and axial components are in about the same proportion as the percentages calculated from the ambient and operating temperature dimension data on page 2-25 of reference 4.

The weighted average hydrogen density at full power was obtained by applying a $\sin^2$ weighting function to the axial density profile through the core. The density profile was constructed from a coolant temperature profile (p. 3-40) and core inlet pressure and pressure drop data (p. 2-2) from reference 6, a NERVA II reactor design study. The average density so obtained occurred on the density profile where the pressure was about 600 psi and the temperature was about 1210° K. In the beryllium reflector, the pressure was 725 psi.

The input for the neutron thermalization code TEMPEST required as input an average moderator temperature. The film temperature (arithmetic average of the graphite surface and bulk coolant temperatures) at the position of the weighted average hydrogen density was chosen as representative. At full power it was 1670° K.

Although the values of coolant temperature and pressure, and the temperature of materials during the power ascension are arbitrary (constrained only by heat-transfer considerations or pump characteristics), a typical power ascension program involves pressure and temperature ramps that are essentially linear. Therefore, with the exception of the reflector, values of these parameters at levels of power less than 100 percent were set proportional to power; that is, in the case of temperature, the difference between the full-power temperature and the base temperature of 293° K, was multiplied by the appropriate percentage of full power and the result added to 293° K. Although the bulk expansion is not an exactly linear function of temperature (the expansion at the temperature for 50-percent power is slightly less than 1 percent), it was similarly treated. Temperatures, densities, coolant pressures, and percentages of bulk expansion in the core at each power level are listed in table III.

Core hydrogen reactivity coefficients ($/kg H_2) were obtained by using room temperature dimensions and atom densities of core and reflector materials in combination with full-power atom densities of hydrogen in the core. The calculations were repeated by using 50 percent of that hydrogen density to detect nonlinearity in the coefficient. A third set of results for a density about 10 percent of that at full power was provided by the data from the calculations at room temperature with coolant.

Core temperature reactivity coefficients ($/$1000^O$ K) were obtained from another set of calculations in which hydrogen was excluded to isolate the effect of temperature. The temperature, atom densities, and reactor dimensions corresponded to those of full-power operation. These calculations were also repeated at an intermediate power to detect nonlinearity.

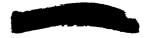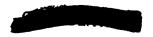The worth of hydrogen in the reflector was evaluated by recalculating k_{eff} using

TABLE III. - TEMPERATURES AND DENSITIES

Power, percent	Temperature, °K			Bulk expansion, percent	Coolant pressure, psi	Coolant density, g/cm³	Void fraction	Atom density, atoms/b-cm	
	Coolant	Moderator in TEMPEST	Structure					Coolant[a]	Structure
Core									
0	293	293	293	0	14.7	0.0000899	0.2	0.0000107	(b)
							.3	.0000161	(b)
							.4	.0000215	(b)
25	520	630	690	0.5	150	0.000489	0.2	0.0000584	(c)
							.3	.0000876	
							.4	.000117	
50	750	980	1090	1.0	300	0.000673	0.2	0.0000804	
							.3	.000121	
							.4	.000161	
75	980	1330	1490	1.5	450	0.000769	0.2	0.0000918	
							.3	.000138	
							.4	.000184	
100	1210	1670	1890	2.0	600	0.000847	0.2	0.000101	
							.3	.000152	
							.4	.000202	
Graphite support annulus									
0	----	293	293	0	-----	---------	0	----------	0.0803
25	----	370	370	.125	-----	---------		----------	.0802
50	----	450	450	.25	-----	---------		----------	.0801
75	----	520	520	.375	-----	---------		----------	.0800
100	----	600	600	.5	-----	---------		----------	.0799
Beryllium reflector									
0	293	293	293	0	14.7	0.0000899	0.175	0.00000940	0.10197
25	150	150	150	-.3	170	.00184		.000192	.1023
50					340	.00368		.000385	
75					510	.00561		.000586	
100					725	.00777		.000812	

[a]Homogenized over volume of region.
[b]See table II.
[c]Use table II and bulk expansion from this table.

full-power parameters, but with hydrogen excluded from the reflector. To verify these results k_{eff} was calculated with the cores at room temperature without hydrogen, but with hydrogen at full-power reflector density in the reflector.
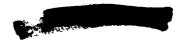
To determine if there was significant coupling between the core and reflector hydrogen effects, k_{eff} was again calculated at room temperature conditions but with hydrogen at full-power density in both core and reflector. The worth of hydrogen in the core was obtained from the calculations of the core hydrogen coefficient in which hydrogen at full-power density was present in the core but excluded from the reflector. The results of the coupling test were negative in that the values of hydrogen worth from the calculations with hydrogen in both core and reflector were nearly identical to the sum of the independently calculated core and reflector worths.

The control-system worth was calculated for the base (room temperature without coolant) and second (room temperature with coolant) cases, and at full power by using a modification of a method devised by Charmatz (ref. 7). After the core size had been determined, a fourth annular region 0.030 centimeter thick and consisting of a material that is a strong neutron absorber (in this case, boron 10 with an atom density of 0.1364 atom/b-cm) was placed between the graphite support annulus and the beryllium reflector. The k_{eff} of the reactor was calculated, and the worth of the absorbing annulus or "curtain" was multiplied by the ratio of the effective surface area of the boron 10 on the control drums to the surface area of the curtain. The curtain-out calculation was not required because its effect in that position is negligible.

The number of drums used for each reactor was based on an arbitrary choice of 50-percent utilization of the circumferential dimension of the reflector calculated at the median reflector radius. This compares to about 40 percent for the 12-drum NERVA I design and 75 percent for the 24-drum NERVA II design. The ratio of control-system worth to curtain worth, however, will be greater than 0.5 because of the boron 10 configuration and the way in which its area is calculated and used (see appendix C) and because of the requirement that the number of drums be integral.

RESULTS AND DISCUSSION

The irregular character of some of the plotted data (e.g., the variation of net reactivity worth with reactor power at the smaller values of moderator to fuel ratio) is believed to be attributable to round off errors in the calculation of the atom densities of graphite and uranium in their expanded, high temperature state for the various power levels. At larger moderator to fuel ratios, where the magnitude of the reactivity effect is greater, the results display better continuity. Connection of calculated points in the figures by straight-line segments rather than smooth curves should be considered, however,

not as an assumption concerning the continuity of the results, but merely as a means of more readily relating results to which particular values of the core void fraction or moderator to fuel ratio are common.

Effect of Moderator to Fuel Ratio and Core Void Fraction on Core Size

The first requirement of this study was the determination of the core size for each of the 20 combinations of moderator to fuel ratio R and core void fraction α. The core diameters are listed in table IV. At a core void fraction of 0.2, the diameters ranged from 61.5 centimeters at $R = 50$ to 147.6 centimeters at $R = 5000$. At a core void fraction of 0.4, the range was from 93.0 centimeters at $R = 50$ to 267.5 centimeters at $R = 2000$.
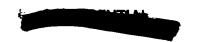
The increase in core size as R increases is primarily a consequence of a shifting neutron energy spectrum. If an undiluted and unreflected cylinder of uranium 235 in the critical state ($k_{eff} = 1$) is initially considered, the neutron spectrum is essentially a fission spectrum. If the uranium 235 is diluted with a moderator such as graphite, the initial increment of dilution shifts the spectrum to a lower energy range. An initial value of R may now be defined. The energy of a larger fraction of the neutron population falls within the range of 1 to 100 electron volts where uranium resonances (narrow energy bands characterized by extremely large cross sections for nonproductive neutron capture) exist, and the increase in nonproductive captures, relative to those that result in fissions, necessitates the addition of more uranium 235 if criticality is to be maintained. The addition of uranium 235 at this fixed initial value of R necessarily involves addition of graphite as well; the resultant increase in reactor size partially offsets the requirement for additional uranium 235 by reducing the leakage. As more graphite is proportionately added, however, the

TABLE IV. - CORE DIAMETERS FOR EFFECTIVE

MULTIPLICATION FACTOR OF 1.05

Core void fraction, α	Moderator to fuel ratio, R	Core diameter, cm	Mass of uranium 235, kg
0.2	50	61.5	193
	100	85.1	187
	200	111.5	161
	500	118.1	72
	1000	115.1	34
	2000	120.4	19
	5000	147.6	11
0.3	50	73.4	240
	100	105.7	251
	200	146.8	243
	500	158.5	114
	1000	153.4	42
	2000	161.5	30
	5000	209.6	20
0.4	50	93.0	330
	100	144.8	405
	200	231.9	520
	500	265.2	273
	1000	249.2	121
	2000	267.5	70

spectrum continues to shift to lower energies. The mass of uranium 235 required to maintain criticality (critical mass) continues to increase until the spectrum has shifted to the low energy side of the resonance range. When the fraction of the neutron population with energy within the resonance range (and the number of resonant, nonproductive neutron absorptions) begins to decrease, the critical mass will begin to decrease. The addition of graphite now contributes to the reduction in critical mass. The critical mass attains its minimum value when sufficient graphite has been added to bring the neutron spectrum into thermal equilibrium with the temperature of the carbon atoms (thermalize the spectrum). If more carbon atoms are added, they serve only to absorb additional neutrons, and the critical mass again increases.

The preceding discussion can also be applied to a family of reflected reactors such as those that are the subject of this report. However, at any given atomic ratio of carbon to uranium 235, the size and critical mass of the reflected core will be smaller than for an unreflected core, and the average neutron energy in the core will be lower. Leakage is substantially reduced because many neutrons that leave the core at epithermal energies are thermalized in the reflector and returned to the core. Consequently, the critical mass variation for reflected cores may or may not show a maximum; and the minimum, corresponding to a thermalized spectrum, may occur at a lower atomic ratio of carbon to uranium 235. The core size and critical mass at the minimum may also be smaller. Despite these differences, however, and with the possible exception of the absence of a maximum in the critical mass variation, the reflected cores will respond to increasing dilution of the fuel in the same way as the bare cores.

The mass of uranium 235 in each core is listed in table IV. At a core void fraction of 0.2, the maximum is either absent or, more probably, occurs at a value of $R < 50$. At core void fractions of 0.3 and 0.4 the maxima are evident. Because a higher core void fraction permits greater leakage (primarily in the axial direction), the higher the core void fraction, the higher will be the value of R at which the maximum occurs. In no case has dilution been carried beyond that necessary for thermalization.

For R greater than that at which the critical mass attains its maximum value, addition of graphite causes the size to increase, although the critical mass is decreasing. The decrease in core diameter with increasing R in the range from 500 to 1000 is probably attributable to inadequate group structure in the GAM II code, as noted by Peoples and Fieno (ref. 1).

Increasing the core void fraction at constant R requires an increase in core diameter and, consequently, an increase in uranium 235 mass. There is, in this case, a rather complex "trade-off" between radial and axial, and thermal and epithermal leakages. Although the effect of a change in α on thermal and epithermal radial leakage is nearly the same (as well as small because of the reflector), leakage in the axial direction is considerably more sensitive to α in the epithermal energy range than in the thermal.

Therefore, although the increase in core diameter that is required to reduce the leakage (and return the value of k_{eff} to 1.05) is effective in reducing the epithermal as well as thermal radial leakage, the increase in epithermal axial leakage is sufficient to produce an increase in the total epithermal leakage. A much smaller proportionate increase in the thermal axial leakage permits the total thermal leakage to decrease and compensates for the effect of epithermal leakage.

Combined Temperature and Coolant Induced Reactivity Effects

Values for the worth of hydrogen to the reactor were obtained with hydrogen at room temperature and 1 atmosphere pressure in the core and reflector. These results are listed in table V as the values at zero power. The worth ranged from a negligible value at $\alpha = 0.2$ and R = 50, to \$0.53 at $\alpha = 0.4$ and R = 1000. The worth tended to increase as either α or R increased.

TABLE V. - NET WORTH OF COOLANT AND TEMPERATURE

EFFECTS TO THE REACTOR

Core void fraction, α	Moderator to fuel ratio, R	Power, percent				
		0	25	50	75	100
		Worth to reactor, \$				
0.2	50	-0.01	-0.07	0.06	-0.05	-0.21
	100	.03	.10	.11	-.03	-.06
	200	.10	.31	.40	.27	.26
	500	.17	.37	.30	.01	-.15
	1000	.20	-.31	-1.03	-1.79	-2.29
	2000	.22	-2.15	-4.19	-5.93	-6.99
	5000	.18	-6.78	-11.96	-15.85	-18.47
0.3	50	0.07	0.05	0.19	0.10	0.20
	100	.07	.24	.26	.31	.24
	200	.18	.72	.84	.93	.87
	500	.30	1.00	1.10	1.10	.98
	1000	.33	.35	-.19	-.58	-1.06
	2000	.33	-1.44	-3.40	-4.73	-5.86
	5000	.24	-6.18	-11.22	-14.76	-17.10
0.4	50	0.10	0.38	0.41	0.41	0.40
	100	.10	.57	.60	.63	.49
	200	.27	1.34	1.59	1.67	1.61
	500	.48	2.07	2.37	2.44	2.29
	1000	.53	1.59	1.22	.93	.42
	2000	.50	-.21	-1.89	-3.25	-4.39

(a) Moderator to fuel ratio, 50.

(b) Moderator to fuel ratio, 100.

(c) Moderator to fuel ratio, 200.

(d) Moderator to fuel ratio, 500.
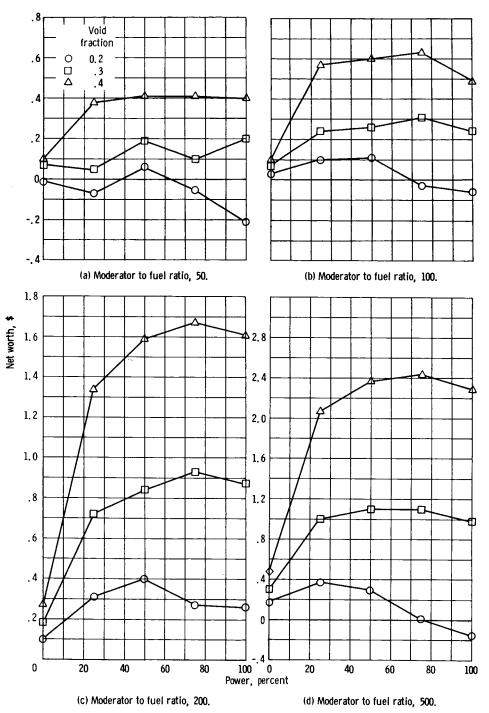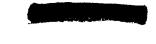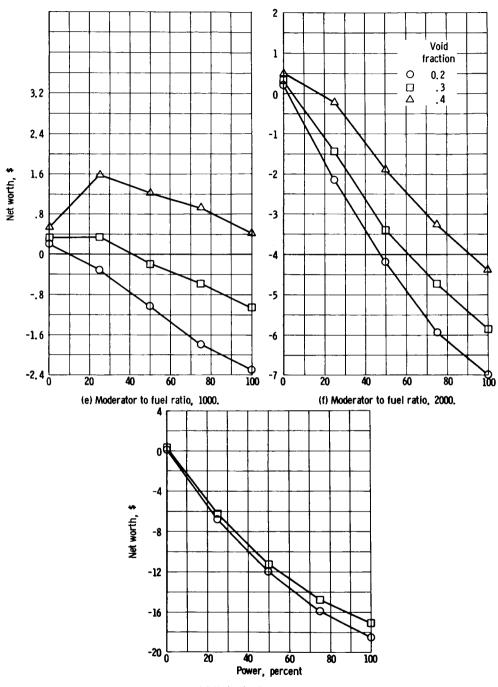
Figure 2. – Combined temperature and coolant induced reactivity effects.

(e) Moderator to fuel ratio, 1000.

(f) Moderator to fuel ratio, 2000.

(g) Moderator to fuel ratio, 5000.

Figure 2. – Concluded.

Because of small atomic mass, hydrogen is an excellent moderator; therefore, the introduction of it into the reactor tends to increase the multiplication factor (pp. 14 and 15). At ambient temperature and pressure, however, the density is only about 10 percent of the full-power density (although the full-power temperature is much higher, the increase in pressure is sufficient to increase the density by a factor of 10), and the worth is small.

The combined net worth to the reactor of hydrogen and temperature in the core, graphite core support annulus, and reflector at 25, 50, 75, and 100 percent of full power are also given in table V. The worth is shown as a function of power in figure 2. At R = 50 the profiles are quite flat ranging from a maximum positive value of $0.41 for $\alpha = 0.4$ at 50- and 75-percent power to -$0.21 for $\alpha = 0.2$ at 100 percent. The effects are small in magnitude because the core hydrogen and temperature reactivity coefficients (see pp. 14, 15, and 16) are rather small, and because the reactors contain relatively little hydrogen. As R increases, the net worth tends to become more positive reaching a maximum of $2.44 at 75 percent for R = 500 and $\alpha = 0.4$. The increasing size of the reactors and the increasing mass of hydrogen are sufficient to overcome the small increase in the magnitude of the negative temperature coefficient despite a core hydrogen reactivity coefficient that decreases for R between 50 and 100 (for $\alpha = 0.2$) or 200 (for $\alpha = 0.4$). At R > 500 the sharply increasing temperature coefficient becomes predominant, and the net worth becomes increasingly negative reaching a value of -$18.47 for R = 5000 and $\alpha = 0.2$.

At all values of R, the highest values of positive net worth and the smallest values of negative net worth occurred with the largest α. This result is consistent with a positive hydrogen coefficient, because the mass of hydrogen in the core is proportional to the core void fraction. Furthermore, for the particular choice of power ascension program that is characterized by the temperatures and pressures used herein, the largest positive values of net worth were found at or below 75-percent power. Temperature effects become predominant by attainment of 50-percent power because the hydrogen gas density has already risen to 80 percent of the full-power density. The remaining increment of density or, in effect, hydrogen mass, has a positive reactivity effect that is small compared to the negative effect of the remaining temperature increment.

Effect of Hydrogen in the Reflector

The dollar worth at full power of reflector hydrogen and the coefficients ($/kg H_2) are listed in table VI. The worth ranged from $0.49 for $\alpha = 0.2$ and R = 50 to about $0.05, regardless of core void fraction, at R = 5000. In general, however, the worth decreased as R or α increased. The coefficients ranged from 0.91 $/kg H_2 for $\alpha = 0.2$ and R = 50 to less than 0.05 $/kg H_2 for R = 5000. They showed the same variation with R and α as did the values of worth.

TABLE VI. - HYDROGEN AND TEMPERATURE REACTIVITY COEFFICIENTS

Core void fraction, α	Moderator to fuel ratio	Temperature range, °K			[a]Hydrogen coefficient, ω_{H_2}, $/kg H_2	Worth of hydrogen in reactor	
		Partial		[a]Full, ω_T		Reactivity worth, $	Hydrogen coefficient, $/kg H_2
		293 to 1190	1190 to 1890	293 to 1890			
		Temperature coefficient, $/1000°K					
0.2	50	-0.52	-0.92	-0.70	4.07	0.49	0.91
	100	-.56	-.77	-.65	3.69	.39	.57
	200	-.65	-.77	-.70	4.44	.29	.34
	500	-1.39	-1.31	-1.37	6.28	.25	.28
	1000	-3.28	-2.27	-2.84	7.09	.22	.25
	2000	-7.36	-4.05	-5.94	6.36	.16	.17
	5000	-17.23	-8.24	-13.36	3.63	.06	.05
0.3	50	-0.58	-0.56	-0.58	4.01	0.45	0.73
	100	-.70	-.61	-.67	2.82	.34	.41
	200	-.77	-.72	-.74	2.97	.23	.21
	500	-1.49	-1.10	-1.33	4.09	.18	.16
	1000	-3.40	-1.98	-2.79	4.66	.16	.14
	2000	-7.56	-3.80	-5.94	4.04	.11	.09
	5000	-17.30	-7.54	-13.09	2.02	.03	.02
0.4	50	-0.61	-0.63	-0.63	2.96	0.40	0.54
	100	-.74	-.85	-.79	1.76	.26	.24
	200	-.76	-.97	-.85	1.41	.13	.08
	500	-1.49	-1.39	-1.46	1.74	.09	.05
	1000	-3.28	-2.39	-2.90	2.10	.08	.05
	2000	-7.36	-4.21	-6.01	1.78	.05	.03
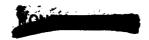
[a]$\omega_{tot} = \omega_{H_2}\left(M_{H_2}\right) + \omega_T \dfrac{(T - 293)}{1000}$ where ω_T is the net reactivity worth to the reactor, M_{H_2} is the mass of hydrogen in the reactor core in kilograms, and T is the core material temperature in °K.
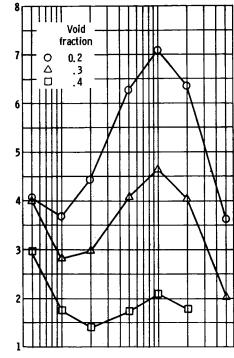
The decrease in worth with increasing R is attributable to the decreasing significance of the return current of thermal neutrons from the reflector into the core as the spectrum shifts to lower energy. Axial leakage may be responsible for the variation with α. As α increases, the axial leakage and the core diameter increase, and fewer epithermal neutrons reach the reflector.

Core Hydrogen Reactivity Coefficient

Results of the calculations to determine the core hydrogen reactivity coefficients ($/kg H_2) are given in table VI and are shown as functions of R in figure 3(a). The coef-
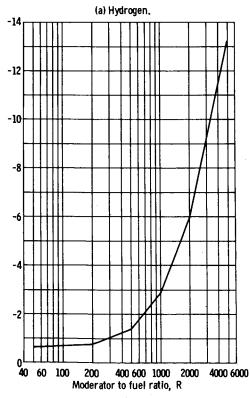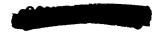
(a) Hydrogen.

(b) Temperature.

Figure 3. - Core-reactivity coefficients as function of moderator to fuel ratio.
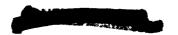
ficients ranged from 1.41 \$/kg H_2 for R = 200 and α = 0.4 to 7.09 \$/kg H_2 for R = 1000 and $\alpha = 0.2$. Three calculations of the coefficients were made for each reactor covering the range of gas density from that at room temperature and 1 atmosphere pressure to that at full power, an increase of about a factor of 10. There were no significant changes over this density range. The values shown were calculated from full-power densities.

At all values of R the coefficients become smaller as α increases. This effect is probably related (as was that of reflector worth) to the increase in leakage (primarily axial) that accompanies the increase in α, thus detracting from the positive effect that the increased mass of hydrogen would otherwise have.

The variation of the hydrogen coefficient with R has a form that suggests an explanation similar to that used to explain the critical mass variation. For R less than about 150, hydrogen may shift the spectrum farther into the resonance region. Although the net effect remains positive because of a reduction in leakage, the increased nonproductive capture in uranium reduces its magnitude. For R > 150, hydrogen may shift the spectrum away from and below the most important resonances. Its effect on leakage would remain unchanged, but nonproductive capture would decrease, and the coefficient would increase. For R > 1000, the relatively fewer atoms of hydrogen compared to the preponderance of carbon atoms may act as additional absorbers in a graphite medium that is very nearly adequate to the task of thermalization in the absence of hydrogen. The coefficient would again become smaller.

Core Temperature Reactivity Coefficient

Core temperature reactivity coefficients ($/1000° K) for the range from room temperature (293° K) to full-power temperature (1890° K), as well as half-range coefficients, are listed in table VI. For R < 500, the difference between the coefficients for the lower and upper range of temperature was small. In some cases, the lower range coefficient was higher, in others the upper range. As R increased above 500, however, the upper range coefficient became a progressively smaller fraction of that for the lower range, and at R = 5000 was less than half as large. The full-range coefficients were averaged and are shown as a function of R in figure 3(b) (the dependence on α was negligible). At R = 50 they are about 0.6 $/1000° K, increase only slightly for 50 < R < 500, and then increase sharply for R > 500 to about 13.2 $/1000° K at R = 5000.
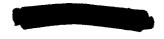
During the power ascension, as the temperature of the core increases, it expands. This reduces the atom densities and thus permits increased leakage, particularly in the axial direction. For R > 1000 where the core diameter to height ratio begins to increase sharply, the increase in cross-section area causes a corresponding increase in axial leakage that is, in turn, at least partly responsible for the increase in the magnitude of the coefficient.
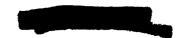
Comparisons of Calculated Reactivity Coefficients

Differences between the reactors included in this parametric study and either the NERVA II reactor or those of the NRX type are necessarily many. They include core zoning (not considered in this study), structure, fuel-element coatings of materials with large cross sections for neutron absorption in the NRX and NERVA II reactors, and small differences in the core parameters R and α. Comparison of the core hydrogen and temperature coefficients calculated here with calculated values reported in references 4 and 6 should, therefore, be considered qualitative.

The NRX-A reactor has a moderator to fuel ratio of about 110 and a core void fraction of 0.29. The core hydrogen and temperature coefficients reported here for R = 100 and α = 0.3 are 2.82 $/kg H_2 and -0.67 $/1000° K, respectively. From the expressions for reactivity worth of hydrogen and temperature given in reference 4 (pp. 11-2 and 11-3), values of 3.09 $/kg H_2 and -1.04 $/1000° K were obtained. The calculated values reported here were lower by 9 and 36 percent, respectively.

The NERVA II reactor has a moderator to fuel ratio of about 170 and a core void fraction of about 0.4. The values of the coefficients reported here for a reactor of this description are about 1.5 $/kg H_2 and -0.85 $/1000° K, respectively. From reference 6 (p. 3-104), if hydrogen in the tie tubes is included, a coefficient of 2.23 $/kg H_2 is ob-

tained. The loss of reactivity caused by temperature and expansion (but not including the Doppler effect) is reported in reference 6 to be about $1.30, yielding a coefficient of $0.81 \$/1000^O$ K. The calculated values reported here are, in this case, 33 percent low and 5 percent high, respectively.

Control-System Worth

Results of the calculations of control-system worth at room temperature conditions and without coolant are listed in table VII. Also listed are the core diameter, curtain worth, number of control drums, and the coolant-flow passage area (a parameter that may be considered proportional to the total power in MW attainable by each reactor if the coolant hole size does not change). Control-system worths ranged from $4 to $16 for systems with from 11 to 40 control drums.

TABLE VII. - CORE DIAMETERS AND CONTROL SYSTEM WORTHS

[Zero power; no coolant.]

Core void fraction, α	Moderator to fuel ratio, R	Core diameter for effective multiplication factor of 1.05		Curtain worth, $	Number of drums	Control system worth, $	Flow passage area	
		in.	cm				m^2	ft^2
0.2	50	24.2	61.5	18.43	11	13.60	0.059	0.64
	100	33.5	85.1	15.59	15	11.80	.113	1.22
	200	43.9	111.5	13.42	18	9.56	.195	2.10
	500	46.5	118.1	16.92	19	12.05	.219	2.36
	1000	45.3	115.1	21.31	19	15.55	.208	2.24
	2000	47.4	120.4	22.69	20	16.72	.228	2.45
	5000	58.1	147.6	18.01	23	12.63	.342	3.68
0.3	50	28.9	73.4	17.46	13	13.08	0.127	1.37
	100	41.6	105.7	13.71	17	9.68	.264	2.84
	200	57.8	146.8	10.39	23	7.31	.509	5.48
	500	62.4	158.5	12.44	25	8.87	.594	6.39
	1000	60.4	153.4	15.83	24	11.18	.556	5.98
	2000	63.6	161.5	16.40	25	11.50	.616	6.63
	5000	82.5	209.6	11.10	32	7.78	1.037	11.16
0.4	50	36.6	93.0	15.64	16	11.68	0.271	2.92
	100	57.0	144.8	10.48	23	7.48	.658	7.08
	200	91.3	231.9	5.71	35	3.97	1.688	18.17
	500	104.4	265.2	5.86	39	3.99	2.207	23.76
	1000	98.1	249.2	7.84	37	5.38	1.949	20.98
	2000	105.3	267.5	7.56	40	5.24	2.245	24.17

If R was held constant, the curtain and control-system worths became smaller as α was increased (in agreement with the results of Peoples and Fieno, ref. 1). If α was held constant and R increased, the curtain and control-system worths decreased between ratios of 50 and 200, reaching a minimum near 200, and then increased to a maximum at about 2000, above which the worth again decreased. This behavior is probably related to the insensitivity of core size to the moderator to fuel ratio in the range from 200 to 1000. It is at variance with the results of reference 1 because the number of control drums used herein was allowed to vary, whereas, in reference 1, the number was held to 12.

Excess of Control System Worth Over Range of Reactivity Variation

The excess of control-system worth (calculated at zero power and without coolant) over the range of the reactivity variation induced by the coolant and high temperature is shown in figure 4 as a function of core diameter and in figure 5 as a function of flow passage area.

The margin between control-system worth and the range of reactivity effects attributable to power operation (shutdown margin) declines sharply as α and the diameter of the
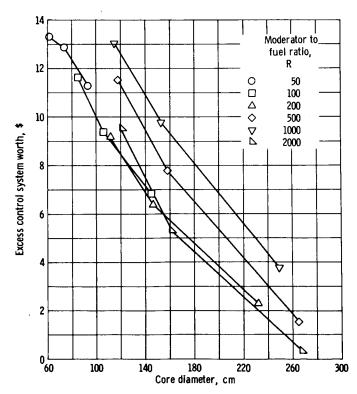

Figure 4. - Excess of control system worth over range of reactivity variation as function of core diameter. The three points for each value of R correspond to core void fractions of 0.2, 0.3, and 0.4, from smaller to larger core diameters, respectively.
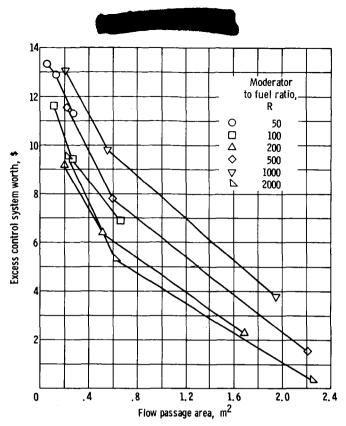
Figure 5. - Excess of control system worth over range of reactivity variation as function of core flow passage area. The three points for each value of R correspond to core void fractions of 0.2, 0.3, and 0.4, from smaller to larger core diameters, respectively.

core become larger. However, if a margin of $2 is considered adequate, only those reactors with very dilute cores (R > 2000) and the two with an α of 0.4 (R = 500 or 2000) must be eliminated from consideration. Over the range of core diameters from 110 to 250 centimeters, the moderator to fuel ratio of 1000 is optimum based on either the worth of the control system or fuel cost. Flow-passage areas as large as 0.9 to 1.4 square meters are feasible.

Effect of Reactor Operation on Control-System Worth

Listed in table VIII are the curtain worth, control-system worth, and the change in control-system worth (from the zero power, no coolant case) for 0 (with coolant) and 100 percent power, respectively. Introduction of hydrogen at room temperature and a pressure of 1 atmosphere had the effect of reducing the control-system worth very slightly (by $0.10 or less). In most cases, the worth increased at full power, the increase ranging up to $4.90 at an α of 0.2 and R of 5000. For R < 1000, however, the increase was less than $1. At a particular value of R, the largest increases occurred at the smallest values of α. If α was held constant and the value of R increased, the
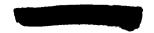
TABLE VIII. - CONTROL SYSTEM WORTH FOR OPERATING CONDITIONS

Core void fraction, α	Moderator to fuel ratio, R	Zero power with coolant			Full power		
		Curtain worth, $	Control system worth, $	Change in control system worth, $	Curtain worth, $	Control system worth, $	Change in control system worth, $
0.2	50	18.35	13.54	-0.06	19.72	14.45	0.85
	100	15.55	11.77	-.03	16.57	12.48	.68
	200	13.38	9.53	-.03	14.06	9.94	.38
	500	16.85	12.00	-.05	17.62	12.46	.41
	1000	21.22	15.48	-.07	22.98	16.66	1.11
	2000	22.58	16.64	-.08	26.64	19.53	2.81
	5000	17.92	12.56	-.07	25.22	17.53	4.90
0.3	50	17.42	13.05	-0.03	18.42	13.70	0.62
	100	13.65	9.64	-.04	14.35	10.04	.36
	200	10.34	7.28	-.03	10.68	7.48	.17
	500	12.35	8.81	-.06	12.53	8.87	0
	1000	15.70	11.08	-.10	16.47	11.55	.37
	2000	16.26	11.40	-.10	18.59	12.96	1.46
	5000	11.02	7.73	-.05	14.95	10.41	2.63
0.4	50	15.57	11.63	-0.05	16.26	12.05	0.37
	100	10.37	7.40	.08	10.78	7.63	.15
	200	5.64	3.92	-.05	5.70	3.93	-.04
	500	5.77	3.93	-.06	5.65	3.82	-.17
	1000	7.72	5.30	-.08	7.79	5.30	-.08
	2000	7.46	5.17	-.07	8.17	5.62	.38

magnitude of the change in control system worth decreased, reaching a minimum for $200 < R < 1000$, and then increased. For $\alpha = 0.4$, a small decrease ($<\$0.20$) was obtained for $200 < R < 1000$.

CONCLUDING REMARKS

A design study of any particular reactor must necessarily apply more sophisticated techniques and consider additional factors than those which are applied and considered in this study. Such a design study might include two-dimensional (to give more adequate treatment to the axial leakage and the axial variation of hydrogen density) diffusion and/or transport calculations, multigroup thermal structure with up- and down-scattering, fuel-element coatings, core zoning, structure of the core-reflector interface, and others.

However, the range of variables covered by a parametric study makes such treatment impractical.

The magnitude of the reactivity perturbation that is attributable to the power ascension and the presence of gaseous hydrogen coolant (propellant) establishes a minimum requirement for the worth of the control system. Within the limitations of core height and reflector design arbitrarily imposed on the family of reactors that were studied, however, this requirement does not seriously restrict the choice of reactor design. By increasing the core height or the thickness of the reflector (ref. 1), or by adding more control drums, design flexibility may be further enhanced. Therefore, although the possibility of building a controllable nuclear rocket reactor with peripheral control and a moderator to fuel ratio much greater than 2000 may be remote, there should be no controllability problem for reactors with diameters up to 180 to 190 centimeters and flow passage areas of 0.9 to 1.4 square meters (10 to 15 ft^2).

Lewis Research Center,
    National Aeronautics and Space Administration,
        Cleveland, Ohio, September 1, 1966,
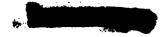   122-29-03-08-22.

APPENDIX A

SYMBOLS

A   atomic or molecular weight

H   core height

k   multiplication factor

k_1 multiplication factor without curtain

k_2 multiplication factor with curtain

M   mass, kg

N   atomic or molecular density, $(b\text{-}cm)^{-1}$

N_0 Avogadro's number

R   moderator to fuel ratio, ratio of total carbon atoms to uranium 235 atoms

R*   moderator to fuel ratio, ratio of total carbon atoms to molecules of uranium dicarbide

r_c core radius

r_d control drum radius

t_s support annulus thickness

v   volume fraction

α core void fraction

β_{eff} effective delayed neutron fraction

ρ material density, g/cm^3

ω_{H_2} hydrogen coefficient of reactivity, $\$/kg\ H_2$

ω_T temperature coefficient of reactivity, $\$/1000^\circ\ K$

ω_t net reactivity worth, $

Subscript:

eff   effective

APPENDIX B

CALCULATION OF ATOM DENSITIES OF CARBON AND URANIUM IN THE CORE

Let

$$v_C + v_{UC_2} + \alpha = 1 \qquad (B1)$$

Carbon is present as graphite and in chemical combination with uranium as uranium dicarbide. Therefore, the total atom density of carbon atoms is

$$N_C + 2N_{UC_2}$$

The moderator to fuel ratio R is defined as

$$\frac{N_C + 2N_{UC_2}}{N_{U-235}}$$

and $R*$ is defined as

$$\frac{N_C + 2N_{UC_2}}{N_{UC_2}}$$

Then
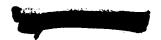
$$R* = \frac{N_C}{N_{UC_2}} + 2 = \frac{v_C \rho_C A_{UC_2}}{v_{UC_2} \rho_{UC_2} A_C} + 2$$

If

$$\rho_C = 1.6 \text{ g/cm}^3$$

$$\rho_{UC_2} = 11.6 \text{ g/cm}^3$$

$$A_C = 12.01$$

$$A_{UC_2} = 259.2$$

$$v_{UC_2} = \frac{\rho_C A_{UC_2} v_C}{\rho_{UC_2} A_C (R^* - 2)} = \frac{2.98 v_C}{R^* - 2} \qquad \text{(B2)}$$

From equation (B1)

$$v_C + \frac{2.98 v_C}{R^* - 2} = 1 - \alpha$$

$$v_C = \frac{(R^* - 2)(1 - \alpha)}{R^* + 0.98}$$

Now,

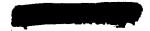$$N_C + 2N_{UC_2} = \frac{v_C \rho_C N_0}{A_C} + 2 \frac{v_{UC_2} \rho_{UC_2} N_0}{A_{UC_2}}$$

From equation (B2)

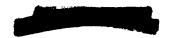$$N_C + 2N_{UC_2} = 0.0803 v_C + 2 \frac{\rho_C A_{UC_2} v_C \rho_{UC_2} N_0}{\rho_{UC_2} A_C (R^* - 2) A_{UC_2}}$$

$$= \frac{0.0803(R^* - 2)(1 - \alpha)}{R^* + 0.98} + \frac{2(0.0803)(R^* - 2)(1 - \alpha)}{(R^* - 2)(R^* + 0.98)}$$

$$= \frac{0.0803(1 - \alpha)}{R^* + 0.98} \left[(R^* - 2) + 2 \right]$$

$$= \frac{0.0803(1 - \alpha)R^*}{R^* + 0.98}$$

Now

$$\frac{R*}{R} = \frac{N_{U-235}}{N_{UC_2}}$$

The uranium is considered to be 93 percent uranium 235 and 7 percent uranium 238. Therefore,

$$R* = 0.93R$$

$$N_{U-235} = 0.93N_{UC_2}$$

$$N_{U-238} = 0.07N_{UC_2}$$

and

$$N_C + 2N_{UC_2} = \frac{0.0747(1 - \alpha)R}{0.93R + 0.98}$$

From the definition of $R*$,

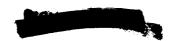$$N_{UC_2} = \frac{N_C + 2N_{UC_2}}{R*}$$

and, thus,

$$N_{U-235} = \frac{0.0747(1 - \alpha)}{0.93R + 0.98}$$

$$N_{U-238} = \frac{0.00562(1 - \alpha)}{0.93R + 0.98}$$

Hydrogen in Core and Reflector

$$N_{H_2} = \frac{v_{H_2}\rho_{H_2}N_0}{A_{H_2}}$$

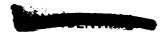But, since hydrogen occupies the voids in the core,

$$v_{H_2} = \alpha$$

and

$$N_{H_2} = \frac{\alpha \rho_{H_2} N_0}{A_{H_2}}$$

If $A_{H_2} = 2.016$, then

$$N_{H_2} = 2.986 \times 10^{23} \, \alpha \rho_{H_2} \, \text{molecules/cm}^3$$

$$N_H = 5.972 \times 10^{23} \, \alpha \rho_{H_2} \, \text{at/cm}^3$$

$$= 0.5972 \, \alpha \rho_{H_2} \, \text{at/b-cm}$$

CALCULATION OF CONTROL DRUM AND REACTIVITY WORTH

The boron 10 annulus (or "curtain") is inserted between the graphite support annulus and the beryllium reflector, and the value of k_{eff} calculated. The worth, in dollars, of the boron 10 annulus to the reactor is

$$\frac{k_1 - k_2}{k_1 k_2 \beta_{eff}}$$

where k_1 is the effective multiplication factor without control drums, and k_2 is the effective multiplication factor from the calculation of k_{eff} with the boron 10 annulus. An effective delayed neutron fraction of 0.0075 has been used throughout this report. This expression is also used to calculate the reactivity worth of changes in composition and temperature. In this case, k_1 and k_2 are the "no control drum" effective multiplication factors for the reactor states being compared. The worth of a control drum is determined by means of a relation devised by Charmatz (ref. 7).

Control drum worth ($) = Curtain worth ($) × $\dfrac{\text{Effective area of boron 10 on drum}}{\text{Area of curtain}}$.

The effective area of boron 10 on the drum is

$$\frac{2\pi}{3}(r_d + 1.5)H$$

where $2\pi/3$ accounts for the 120° azimuthal extent of the boron 10 coating on the drum, and 1.5 centimeters is the approximate thermal neutron mean free path in beryllium. The area of the curtain is

$$2\pi(r_c + t_s)H$$

REFERENCES

1. Peoples, John A.; and Fieno, Daniel B.: Parametric Study of Peripheral Control Effectiveness on Large Core Rocket Reactors. NASA TM X-1244, 1965.

2. Joanou, G. D.; and Dudek, J. S.: GAM II. A B_3 Code for the Calculation of Fast-Neutron Spectra and Associated Multigroup Constants. Rept. No. GA-4265, General Dynamics Corp., Sept. 16, 1963.

3. Shudde, R. H.; and Dyer, J.: TEMPEST - A Neutron Thermalization Code. Atomics International, North American Aviation, Inc., June 1962. (AEC Rep. No. TID-18284.)

4. Staff of Astronuclear Laboratory: Reactor Analysis of NRX-A. Vol. I - Nuclear Analysis. Rept. No. WANL-TNR-128, Westinghouse Electric Corp., Sept. 1963.

5. Keys, R. D.; Kiefer, T. F.; Osgood, S. H.; and Schwartzenberg, F. R.: Cryogenic Materials Data Handbook. (AFML-TDR-64-280, DDC No. AD-609562), Martin Company, Aug. 1964.

6. Staff of Astronuclear Laboratory: Nerva II Reactor Design Evaluations. Rept. No. WANL-TME-1200. Westinghouse Electric Corp., July 1965.

7. Charmatz, Albert W.: Rover Reactor Control Element Worth Calculations. Conference on Nuclear Propulsion, Monterey, California, Aug. 16, 1962. AEC Rept. No. TID-7653 (pt. II), pp. 56-65.